MATH 321: HOMEWORK 9
DUE THURSDAY, DEC 3 BY 11:59PM

Problem 1. Let A be a set with m elements and B be a set with n elements. Count the following.

(1) The number of functions $f : A \to B$.
(2) The number of constant functions $f : A \to B$.
(3) The number of bijections $f : A \to A$.

Problem 2. Check that the function $f : \mathbb{R} \to (0, \infty)$ defined as $f(x) = e^x$ is one-to-one and onto $(0, \infty)$. Check that $f(x + y) = f(x)f(y)$.[1] Determine f^{-1}.

Problem 3. Suppose $f : A \to B$. Prove the following.

(1) If there is a function $g : B \to A$ so that $g \circ f = \mathrm{id}_A$ then f is one-to-one.
(2) If there is a function $g : B \to A$ so that $f \circ g = \mathrm{id}_B$ then f is onto.
(3) If there is a function $g : B \to A$ so that both $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$ then f is a bijection.

[1] Because it has these properties we call f a *group isomorphism* between the groups $(\mathbb{R}, +)$ and $((0, \infty), \cdot)$.