MATH 321: HOMEWORK 8
DUE TUESDAY, NOV 24 BY 11:59PM

For your proofs, you should submit fully written up formal proofs, i.e. not scratchwork.

Recall that a nonstrict order is a reflexive, transitive, antisymmetric relation and a strict order is an antireflexive, transitive, antisymmetric relation. (See slides on the course website for definitions.)

Problem 1. For the following relations, say whether they are a nonstrict order, a strict order, or neither. Justify your answers. Throughout assume that the set X has at least 3 elements.

(1) The equality relation $=$ on a set X.
(2) The empty relation $E = \emptyset$ on a set X—that is, $x \mathrel{E} y$ is always false for all $x, y \in X$.
(3) The indiscrete relation $I = X \times X$ on a set X—that is, $x \mathrel{I} y$ is always true for all $x, y \in X$.

Problem 2. Show that there is a correspondence between strict and nonstrict orders. Namely, show the following two facts:

(1) If $\sqsubset$ is a strict order on A then $\sqsubseteq = \{(a, b) \in A : a \sqsubset b \text{ or } a = b\}$ is a nonstrict order on A.
(2) If $\sqsubseteq$ is a nonstrict order on A then $\sqsubset = \{(a, b) \in A : a \sqsubseteq b \text{ and } a \neq b\}$ is a strict order on A.

Recall that an equivalence relation is a reflexive, symmetric, transitive relation.

Problem 3. For the following relations, say whether or not they are reflexive, symmetric, and/or transitive. Justify your answers. Throughout assume that the set X has at least 3 elements.

(1) The inequality relation $\neq$ on a set X.
(2) The empty relation $E = \emptyset$ on a set X—that is, $x \mathrel{E} y$ is always false for all $x, y \in X$.
(3) The indiscrete relation $I = X \times X$ on a set X—that is, $x \mathrel{I} y$ is always true for all $x, y \in X$.

Problem 4. Suppose $f : A \to B$ is a function with A as a domain. Define a relation $\sim$ on A as: $a \sim a'$ iff $f(a) = f(a')$. Show that $\sim$ is an equivalence relation on A.

Use this result to explain why $\equiv \mod n$ is an equivalence relation on $\mathbb{Z}$, for every $n \geq 1$.