MATH 321: HOMEWORK 7
DUE THURSDAY, NOV 5 BY 11:59PM

For your proofs, you should submit fully written up formal proofs, i.e. not scratchwork.

Problem 1. Do problem #1 from page 295 of the textbook.

Take the following recursive definition of exponentiation with natural number exponents:

- $x^0 = 1$ for all x;
- $x^{n+1} = x^n \cdot x$ for all x and all $n \in \mathbb{N}$.

Problem 2. Use induction and this definition of exponentiation to prove the following rules for exponentiation:

(1) $x^n \cdot y^n = (xy)^n$ for all x, y and all $n \in \mathbb{N}$.
(2) $x^n \cdot x^m = x^{n+m}$ for all x and all $n, m \in \mathbb{N}$.
(3) $(x^n)^m = x^{nm}$ for all x and all $n, m \in \mathbb{N}$.

[Hint: for 2 and 3 there are two natural number variables n and m in the statement of the fact. Which one do you want to do induction on?

Problem 3. Use strong induction to prove that every natural number can be written in binary as a sum of distinct powers of 2. That is, prove that for all n you can write

$$n = 2^{k_1} + 2^{k_2} + \cdots + 2^{k_\ell},$$

where $k_1 > k_2 > \cdots > k_\ell$ are natural numbers.