

**MATH 321: HOMEWORK 5**  
**DUE THURSDAY, OCT 8 BY 11:59PM**

For your proofs, you should submit fully written up formal proofs, i.e. not scratchwork.

**Problem 1.** Do problem #12 from page 144 of the textbook.

**Problem 2.** Suppose  $A$ ,  $B$ , and  $C$  are sets. Prove that  $A \cup C \subseteq B \cup C$  if and only if  $A \setminus C \subseteq B \setminus C$ .

**Problem 3.** Use induction to prove: for all natural numbers  $n$ , either  $n$  is even or  $n$  is odd. [To clarify: by even I mean multiple of 2 and by odd I mean 1 plus a multiple of 2. That is, don't just take odd to be defined as "not even" and make it a triviality.]

**Problem 4.** Prove the following stronger form of Euclid's Lemma.

**Lemma.** *Suppose  $a_1, a_2, \dots, a_n$  is a finite list of positive integers and let  $p$  be a prime number. Set  $A = a_1 a_2 \cdots a_n$  to be the product of the  $a_i$ . If  $p \mid A$ , then  $p \mid a_i$  for some  $i$ .*

[Hint: Use the version of Euclid's Lemma from an earlier homework and then do induction on  $n$ .]