

**MATH 321: HOMEWORK 4**  
**DUE THURSDAY, OCT 1 BY 11:59PM**

**Problem 1.** Do Section 3.2 Exercise 12 from the textbook (page 107).

For the following three problems, please do the following. First, give an outline of the steps your proofs will follow. At each step, what are your knowns and what are your goals? Second, write your proofs as paragraphs of mathematical English.

**Problem 2.** Let  $a$  and  $b$  be positive real numbers. Prove that if  $a < 1/a$  and  $b > 1/b$  then  $a < b$ .

**Problem 3.** Let  $a$  and  $b$  be natural numbers. Prove that if  $\gcd(a, b) < a$  and  $\gcd(a, b) < b$  then  $a \nmid b$  and  $b \nmid a$ . [Hint: do proof by contradiction.]

**Problem 4.** Prove that  $\sqrt{5}$  is irrational. [Hint: do proof by contradiction.]