Math 243 Practice Final Exam

December 8, 2020

Name: ______________________________

This is a practice version of the the second midterm. Below are the instructions for the midterm, and the formulae you are given:
This is the final exam. It has 9 problems for a total of 100 points.
Do not consult with other people or with outside references during the exam.
Do not use a calculator or other such electronic device.
Carefully read each question and understand what is being asked before you start to solve the problem. In particular, some questions will ask you to set up an integral but not solve it. **Do not evaluate the integrals if not asked to; that is a great way to waste your time.** Similarly, if you are not asked to fully simplify your answers, then it is okay to not do so. On the other hand, if you are asked to solve an integral or to fully simplify your answers, then do so. Please show all your work and circle or mark in some way your final answers.
After you finish, upload your exam to the Laulima site. Full instructions can be found on the Laulima assignment for the exam.

$$D = f_{xx}f_{yy} - {f_{xy}}^2$$

$$D_{\vec{u}}f = \nabla f \cdot \vec{u}$$

$$\kappa = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$\int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$

$$\int_a^b \sqrt{1 + (y'(x))^2}\ \mathrm{d}x$$

$$\int_a^b \sqrt{(y'(t))^2 + (x'(t))^2}\ \mathrm{d}t$$

$$\int_a^b \sqrt{(r(\theta))^2 + (r'(\theta))^2}\ \mathrm{d}\theta$$

$$\int_a^b y(t)x'(t)\ \mathrm{d}t$$

$$\frac{1}{2}\int_a^b (r(\theta))^2\ \mathrm{d}\theta$$

1. Consider the vectors $\vec{a} = \langle -3, 4, -5 \rangle$ and $\vec{b} = \langle 0, 2, -4 \rangle$. Compute $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$.

2. Consider the curve given by the vector function $\vec{r}(t) = \langle t^2, e^t, t^3 - t$, where $0 \leq t \leq 2$. Compute $\vec{r}'(t)$, $\vec{r}''(t)$, and $\int_0^2 \vec{r}(t)\ \mathrm{d}t$.

3. Consider the curve from the previous problem. Set up but do not solve an integral to give its arc length.

4. Consider the curve from the previous problem. Calculate its curvature at the point $(1, e, 0)$.

5. Consider the curve from the previous problem. Find an equation for the line which passes through the two endpoints of the curve.

6. Consider the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. Set up but do not solve a polar integral to find the area of this triangle.

7. Consider the line segment from $(1, 0)$ to $(0, 1)$. Set up but do not solve a polar integral to find the arc length of this line segment.

8. Consider the curve given by the equation $y = x^3 - x$, where $-1 \leq x \leq 1$. Set up but do not solve an integral to give the arc length of this curve.

9. Consider the surface given by the equation $z = e^x - e^{xy}$. Find an equation for the tangent plane where $(x, y) = (1, -1)$.

10. Consider the function $f(x, y) = e^{x^2}(y^2 + 1)$. Compute all second partial derivatives of $f(x, y)$.

11. Consider the function from the previous problem. Find all local minimums of the function, and show that it has no local maximum.

12. Consider the function $f(x, y) = \sin(x+y)+\cos(x)$. Compute ∇f and determine the directional derivative of $f(x, y)$ at the point $(\pi/4, -\pi/2)$ in the direction of the vector $(1, \sqrt{3})$.

13. Suppose that $z = x^2y^3 + x^3 + y^2$ is a function of x and y, where $x = \sqrt{t}$ and $y = 1/t$ are functions of t. Compute $\frac{\mathrm{d}z}{\mathrm{d}t}$.