Math 243 Practice Midterm 2

November 12, 2020

Name: ____

This is a practice version of the the second midterm. Below are the instructions for the midterm, and the formulae you are given:
Do not consult with other people or with outside references during the exam.
Do not use a calculator or other such electronic device.
Carefully read each question and understand what is being asked before you start to solve the problem. In particular, some questions will ask you to set up an integral but not solve it. **Do not evaluate the integrals if not asked to; that is a great way to waste your time.** Please show all your work and circle or mark in some way your final answers.
After you finish, upload your exam to the Laulima site. Full instructions can be found on the Laulima assignment for the exam.

$$L = \int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$

$$\kappa = \frac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$$

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

1. Compute the cross product of $\vec{a} = \langle -1, 0, 1 \rangle$ and $\vec{b} = \langle 3, 2, -1 \rangle$. Find the unit vector in the same direction as this cross product.

2. $\vec{a}$ and $\vec{b}$ are non-parallel vectors in the xy-plane where $|\vec{a}| = 4$ and $|\vec{b}| = 3$. Find $\vec{a} \times \vec{b}$.

3. Consider the curve given by the vector function $\vec{r}(t) = \langle \sqrt{t}, t^2, t \rangle$, from the point $(0, 0, 0)$ to $(1, 1, 1)$. Set up an integral to calculate the arc length of this curve. **Do not solve the integral.**

4. Consider the plane which intersects the z-axis at the point $(0, 0, 1)$ and contains the line given by the vector equation $\vec{r}(t) = \langle 1, 2, 0 \rangle t + \langle 2, 1, 3 \rangle$. Find an equation for this plane.

5. Consider the plane given by the equation $2x - 3y + z = 5$. Find a vector function for the line which is orthogonal to this plane and which passes through the point $(-1, 3, 4)$.

6. Consider the curve given by the vector function $\vec{r}(t) = \langle 3t, 2t^2, t^3 \rangle$. Calculate the curvature of the curve at the point $(3, 2, 1)$.

7. Find the domain of the the function

$$f(x, y) = \frac{\ln(x - y^2)}{1 + x^2}$$

and sketch a picture of the domain.

8. Consider the function

$$f(x, y) = \frac{(1 - x)(1 - y)}{1 - x - y}.$$

Find the following limit, or explain why it does not exist:

$$\lim_{(x,y) \to (0,0)} f(x, y).$$

9. Consider the same $f(x, y)$ from the previous problem. Find the following limit, or explain why it does not exist:

$$\lim_{(x,y) \to (1,0)} f(x, y).$$

10. Compute all second partial derivatives of $f(x, y) = e^{x^2 + y^2}$.

11. Find an equation for the plane tangent to the surface $z = \cos(x - y) - \sin(x + y)$ at the point $(\frac{\pi}{2}, \frac{\pi}{2}, 1)$.