Math 243 Midterm 1

September 22, 2020

Name: _____

This is practice exam for the first midterm.
Carefully read each question and understand what is being asked before you start to solve the problem. In particular, some questions will ask you to set up an integral but not solve it. **Do not evaluate the integrals if not asked to; that is a great way to waste your time.** Please show all your work and circle or mark in some way your final answers.

$$\int_a^b \sqrt{1 + (y'(x))^2}\ \mathrm{d}x$$

$$\int_a^b \sqrt{(y'(t))^2 + (x'(t))^2}\ \mathrm{d}t$$

$$\int_a^b \sqrt{(r(\theta))^2 + (r'(\theta))^2}\ \mathrm{d}\theta$$

$$\int_a^b y(t)x'(t)\ \mathrm{d}t$$

$$\frac{1}{2}\int_a^b (r(\theta))^2\ \mathrm{d}\theta$$

1. Consider the inequalities

$$x^2 + y^2 + (z-1)^2 \leq 4 \quad \text{and} \quad z \geq 1$$

which describes a region in $\mathbb{R}^3$. Sketch a picture of the region and describe the region in words.

2. Consider the following three points in three dimensional space.

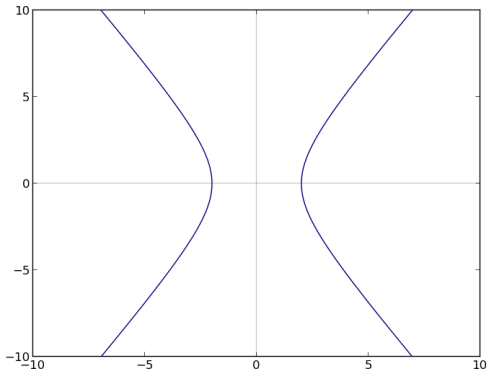
$$P = (1, 0, 3) \qquad Q = (3, 2, -1) \qquad R = (4, 1, 1).$$

Calculate the distances of the line segments between the three points. Which pair of points are the closest together?

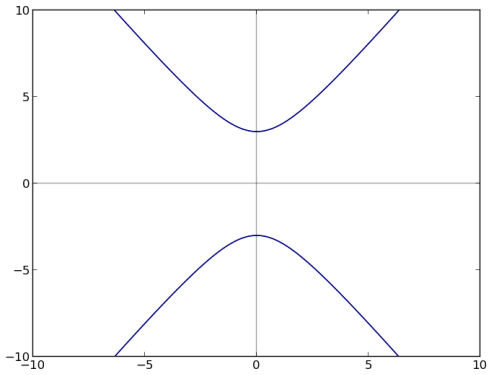
3. Consider the following equation which defines a conic section in the plane.

$$-9x^2 = 4y^2 - 36$$

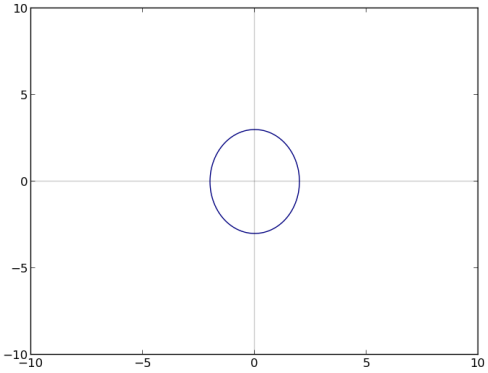
Determine the foci and vertices of this conic section. Which of the following is a graph of this conic section?



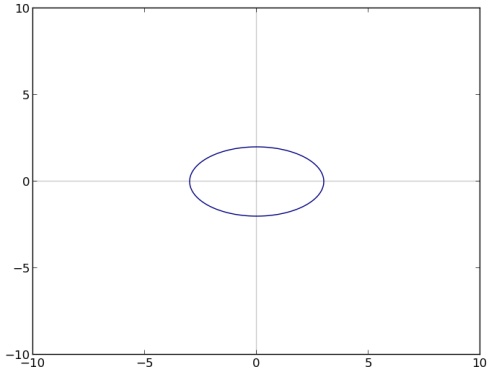
(a)



(b)



(c)



(d)

4. Consider the curve given by the following polar equation

$$r = 1 + \cos\theta$$

Set up a polar integral to calculate the area contained within this curve. Simplify the integrand but **do not calculate the integral**.

5. Consider the following vectors

$$\vec{a} = \langle 1, -2, 3 \rangle, \qquad \vec{b} = \langle 2, 0, -2 \rangle$$

(a) Calculate $|\vec{b}|$ and $|\vec{a} + \vec{b}|$.

(b) Calculate $\vec{a} \cdot \vec{b}$. Based on your answer, determine whether $\vec{a}$ and $\vec{b}$ are parallel, orthogonal, or neither. Explain your answer.

6. Consider the function $f(x) = \ln(x) + \ln(1 - x)$.

   (a) Determine the domain of f.

   (b) Set up an integral to calculate the arc length of the curve given by $f(x)$ over its entire domain. Simplify the integrand as much as possible **but do not actually compute it**.

7. Consider the curve given by the cartesian equation $x^2 + y^2 = 2x$, where it goes from the point $(1, 1)$ to the point $(2, 0)$.

   (a) Convert this cartesian equation to a polar equation.

   (b) Set up a polar integral which gives the arc length of this curve. **Do not calculate the integral.**

8. Determine equations which give the following two conic sections.

   (a) The hyperbola with vertices $(0, \pm 3)$ and foci $(0, \pm 5)$.

   (b) The hyperbola with vertices $(0, \pm 3)$ and foci $(0, \pm 10)$.

   (c) The ellipse with vertices $(0, \pm 3)$ and foci $(0, \pm 1)$.

9. Consider the curve given by the following parametric equations.

$$x = t^2$$
$$y = t^3$$
$$0 \leq t \leq 4$$

(a) Calculate the slope of this curve at the point $(1, 1)$.

(b) Setup an integral which gives the area between this curve and the x-axis. Simplify the integrand as much as possible. State which method you would use to compute this integral **but do not actually compute it**.

10. Consider the curve given by the following parametric equations.

$$x = e^t$$
$$y = t^2$$
$$1 \leq t \leq \ln 4$$

Eliminate the parameter to get a cartesian equation which describes the curve. What are the two endpoints of the curve? Set up a cartesian integral to calculate the arc length of the curve **but do not actually compute it**.