MATH454 HOMEWORK 8
DUE THURSDAY, OCTOBER 24

Exercise 1. Let (X, d) be a metric space. Let τ consist of $\emptyset$ and the subsets of X which are a union of balls $B_r(x) = \{y \in X : d(x, y) < r\}$. Show that τ is a topology on X.

Exercise 2. Suppose (X, d) is a metric space with the metric topology τ. Show that given any two distinct points $x \neq y$ in X that there are open sets $U, V \in \tau$ so that $x \in U$, $y \in V$, and $U \cap V = \emptyset$.

Exercise 3. Consider an arbitrary set X so that $|X| \geq 2$. Let $\tau = \mathcal{P}(X)$ be the *discrete* topology on X and let $\sigma = \{\emptyset, X\}$ be the *indiscrete* topology. Show that there is a metric on X so that τ is the metric topology but there is no metric on X so that σ is the metric topology. [Hint: to show σ cannot come from a metric, use the previous exercise.]

Exercise 4. Do Exercise 5.1 from the textbook. (page 91)

Exercise 5. Do Exercise 5.2 from the textbook.

Exercise 6. Do Exercise 5.3 from the textbook.

Exercise 7 (Reach). Do Exercise 5.4 from the textbook.

Exercise 8 (Reach). Do Exercise 5.5 from the textbook.