Math 244 Final Exam

Monday, December 10

Name: ____

This is the final exam. There are 12 questions, worth a total of 100 points. **No electronic devices are permitted.** Carefully read each question and understand what is being asked before you start to solve the problem. Please show all your work and circle or mark in some way your final answers.

$$\oint_C \vec{F} \cdot \vec{T}\,\mathrm{d}s = \iint_D \operatorname{curl} \vec{F} \cdot \vec{k}\,\mathrm{d}A$$

$$\oint_C \vec{F} \cdot \vec{n}\,\mathrm{d}s = \iint_D \operatorname{div} \vec{F}\,\mathrm{d}A$$

$$\oint_C \vec{F} \cdot \,\mathrm{d}\vec{r} = \iint_S \operatorname{curl} \vec{F} \cdot \vec{n}\,\mathrm{d}\sigma$$

$$\iint_S \vec{F} \cdot \vec{n}\,\mathrm{d}\sigma = \iiint_R \operatorname{div} \vec{F}\,\mathrm{d}V$$

$$\mathrm{d}s = |\vec{r}\,'(t)|\,\mathrm{d}t$$

$$\mathrm{d}\sigma = |\vec{r}_u(u,v) \times \vec{r}_v(u,v)|\,\mathrm{d}u\,\mathrm{d}v$$

1. (8 points) Set up **but don't solve** integrals in spherical and cylindrical coordinates to compute

$$\iiint_B zx + zy \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z,$$

where B is the solid unit ball $x^2 + y^2 + z^2 \leq 1$.

2. (8 points) Set up a line integral which is equal to the surface integral

$$\iint_S \nabla \times (x\vec{\imath} + xy\vec{\jmath} + xyz\vec{k}) \cdot \vec{n}\ \mathrm{d}\sigma$$

where S is the upper hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$. Then set up **but don't solve** an equivalent one-dimensional integral with respect to t.

3. (8 points) Compute the integral

$$\iint_R 10x^2y \,\mathrm{d}x\,\mathrm{d}y$$

where R is the triangle bounded by the lines $x = 1$, $x + y = 1$, and $y - 2x = 1$.

4. (8 points) Compute the integral

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}$$

where $\vec{F} = y\vec{\imath} + x\vec{\jmath}$ and C is the curve parameterized by $\vec{r}(t) = t^7\vec{\imath} + t^{11}\vec{\jmath}$ with $0 \leq t \leq 1$.

5. (10 points) Use the coordinate transform $x = u^2$, $y = v$ to compute the integral

$$\int_0^{\sqrt{15}} \int_0^{\sqrt{x}} 4y\sqrt{x^2+1}\,\mathrm{d}y\,\mathrm{d}x.$$

6. (8 points) Set up **but don't solve** a double integral which is equal to the path integral

$$\int_C (xy\vec{\imath} + e^{xy}\vec{\jmath}) \cdot \mathrm{d}\vec{r}$$

where C is the counterclockwise-oriented square with corners $(-2, -1)$, $(1, -1)$, $(1, 3)$, and $(-2, 3)$.

7. (8 points) The *Laplacian* of a scalar-valued function $f(x, y)$ is $\nabla \cdot \nabla f$. A function $f(x, y)$ is *harmonic* if its Laplacian is 0. Show that $f(x, y) = \log(x^2 + y^2)$ is harmonic.

8. (10 points) Compute the outward flux of the vector field $\vec{F} = (x^3, y^3, -2z^3)$ on the boundary of the cube $0 \leq x, y, z \leq 1$.

9. (8 points) Write an equation for the plane which contains the three points $(0, 1, 1)$, $(0, 0, 0)$, and $(2, -1, -1)$.

10. (8 points) Show that

$$\int_C \left((e^x \sin y + 3x^2)\vec{\imath} + (e^x \cos y + 4y^3)\vec{\jmath}\right) \cdot \, \mathrm{d}\vec{r} = 0$$

where C is the counterclockwise-oriented regular heptakaidecagon (17-gon) centered at the origin with one vertex on the point $(1, 0)$. Explain in words why your calculations show that this integral is 0. (Hint: you don't need to directly calculate the integral.)

11. (8 points) Set up **but don't solve** an integral in polar coordinates to find the area of the intersection of the two circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$.

12. (8 points) Use a triple integral to calculate the volume of the tetrahedron bounded by the three coordinate axes and the plane $z = 2 - x - 2y$.

(Extra space. Please clearly indicate which problem the work is for.)