Math 244 Midterm 1: In-class portion

Monday, September 24

Name: ______________________________________

This is the in-class portion of the first midterm. It is worth 80 points, with the take-home portion being worth 20 points for a total of 100 points. When you are finished with the in-class portion of the exam please bring it to be to receive the take-home portion, which will be due Wednseday, September 26.

No electronic devices are permitted. Carefully read each question and understand what is being asked before you start to solve the problem. Please show all your work and circle or mark in some way your final answers.

1. (5 points) State the version of Fubini's theorem for integrating a function $f(x, y)$ over a rectangular region R given by $a \leq x \leq b$ and $c \leq y \leq d$. (Hint: remember that you need to make an assumption about $f(x, y)$.)

2. (15 points) Set up **but do not solve** two different iterated integrals equal to the double integral

$$\iint_R \frac{e^{xy}}{x} \, \mathrm{d}A,$$

where R is the quarter circle of radius 17 in the first quadrant. One of these integrals must be in rectangular form, the other must be in polar form. (Hint: $17^2 = 289$.)

3. (20 points) Consider the double integral $\iint_R \sin(x - y)\,\mathrm{d}A$ where R is the triangle bounded by the lines $y = x$, $x = \pi/2$, and $y = 0$. Set up two different iterated integrals to calculate this double integral, one where you integrate with respect to x and then with respect to y, and the other where you integrate with respect to y and then with respect to x. Solve one of these two—your choice!—integrals.

4. (10 points) Calculate the double integral

$$\iint_R xy + 4\,\mathrm{d}A$$

over the rectangle R given by $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$.

5. (10 points) Consider the point $P = (0, 2, -1)$ and the line given by $\vec{\ell}(t) = (5\vec{\imath} - \vec{\jmath} + 2\vec{k})t$. Find an equation for the plane determined by the point P and the line $\vec{\ell}(t)$.

6. (20 points) Let R be the region bounded by the curve $xy = 1$ and the lines $y = 0$, $x = e$, and $x = e^3$. Sketch the region R and calculate the double integral

$$\iint_R \sinh(xy)\,\mathrm{d}A.$$

(Hint: $\sinh(u) = \frac{e^u - e^{-u}}{2}$ is the *hyperbolic sine*.)

Math 244 Midterm 1: Take-home portion

Monday, September 24

Name: ____

This is the take-home portion of the first midterm. It is worth 20 points, with the in-class portion being worth 80 points for a total of 100 points. **This is due Wednseday, September 26.**
All work here is to be your own. **Do not ask your fellow students nor the internet for help.** It is okay to use your calculator or a computer algebra system to help with calculations. But if you do, then clearly state where and how you used it.
Carefully read each question and understand what is being asked before you start to solve the problem. Please show all your work and circle or mark in some way your final answers.

1. (6 points) Consider the hyperbolic paraboloid given by the equation

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2},$$

where a and b are positive real numbers. What is the average height of this hyperbolic paraboloid over the rectangular region R given by $0 \leq x \leq a$ and $0 \leq y \leq b$?

2. (6 points) Let $m < M$ be real numbers. What is the average height of the same hyperbolic paraboloid from the previous question over the rectangular region R given by $ma \leq x \leq Ma$ and $mb \leq y \leq Mb$? (So the previous question was the special case where $m = 0$ and $M = 1$. You can check your work by checking that the answer you get here matches the previous answer for this special case.)

3. (8 points) Let $0 \leq m < M$ be non-negative real numbers. Consider the hyperbolic paraboloid given by the equation

$$z = y^2 - x^2.$$

What is the average height of this hyperbolic paraboloid on the annulus region R consisting of all points whose distance to the origin is between m and M? (Hint: recall that $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$.)