WORKSHEET 9

(1) In the following problems, you should write the partial fraction decomposition for the rational function, but do not solve for the constants. For example, for $\frac{x}{(x^2-1)(x^2+1)}$, you should write

$$\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2+1}$$

because the denominator factors as $(x-1)(x+1)(x^2+1)$.

(a) $\dfrac{x-2}{x^2+4x}$

(b) $\dfrac{x^2}{x-1}$ (Be careful! The numerator has higher degree than the denominator.)

(c) $\dfrac{x^2+1}{x^2-9x-10}$

(d) $\dfrac{x^2+2}{x^2(x^2+1)^2(x-3)^3}$

(2) Put the following into partial fractions form:

(a) $\dfrac{2x^2 - 5}{x^2 - 3x - 4}$

(b) $\dfrac{3x + 1}{x^3 + 2x^2 + 4x}$

(3) Integrate the following functions (no need to be put in partial fractions form):

(a) $\displaystyle\int \frac{1}{y^2 - 2y + 4}\mathrm{d}y$

(b) $\displaystyle\int \frac{3x + 2}{2x^2 - 4x + 8}\mathrm{d}x$

(4) Put the following functions in partial fractions form and integrate

(a) $\displaystyle\int_4^8 \frac{y}{y^2 - 2y - 3} \mathrm{d}y$

(b) $\displaystyle\int \frac{x^2 + x}{x^3 + x} \mathrm{d}x$

(5) Find an upper bound for the Trapezoid rule error with $n = 4$ when approximating

$$\int_{-1}^{1} e^{-x^2}$$

You don't need to find the approximation, only the upper bound for the estimate. You may find this helpful:

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}, \quad \text{where } |f''(x)| \leq M \text{ for all } x \text{ in } [a, b]$$

(6) Find an n that would guarantee that Simpson's Rule S_n is within 10^{-8} of $\int_1^4 x^{3/2}\mathrm{d}x$. You may leave your answer abstractly with roots. You can use the following bit of information $\frac{d^4}{dx^4}(x^{3/2}) = \frac{9}{16}x^{-5/2}$. **Again: you don't need to find the approximation.** You may find this helpful:

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}, \quad \text{where } |f^{(4)}(x)| \leq M \text{ for all } x \text{ in } [a, b]$$

(7) For the following problems explain why the integral is improper. Evaluate or explain why it is divergent.

(a) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{x^{3/2}}$

(b) $\displaystyle\int_{-\infty}^0 \theta e^{-\theta}\mathrm{d}\theta$

(8) For the following problems, explain whether the integral is divergent or convergent. You do not have to evaluate. Try to use the p-test if you can.

(a) $\displaystyle\int_1^\infty \frac{1+\sin x}{x^2}\,\mathrm{d}x$

(b) $\displaystyle\int_1^\infty \frac{\sqrt{x^4+1}}{x^3}\,\mathrm{d}x$

(c) $\displaystyle\int_1^\infty \frac{\arctan x}{x}\,\mathrm{d}x$

WORKSHEET 10

(1) For the following sequences, evaluate the limit:

(a) $\{\frac{2n}{5n-1}\}$

(b) $\{\sqrt[n]{n}\}$

(c) $\{0, 1, 0, 1, 0, 1, \dots\}$

(d) $\{\frac{(n-1)!}{n!}\}$

(e) $\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\}$

(f) $a_n = \frac{(\ln n)^2}{n}$

(g) $a_0 = 100,$ and $a_n = \frac{1}{2}a_{n-1}$

(2) For each series, write the first four terms of the sum. Then evaluate the series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{4^n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right)$

(c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

(d) $e^{-1} + e^{-2} + e^{-3} + \dots$

(3) Consider the following 'proof' that $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ converges and its sum is 0:

(a) Use rules of logarithms to simplify the expression

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} \big(\ln n - \ln(n+1)\big)$$

(b)

$$\sum_{n=1}^{\infty} \big(\ln n - \ln n + 1\big) = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + (\ln 4 - \ln 5) + ...$$

(c) Cancel repeated terms in the series:

$$(\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + (\ln 4 - \ln 5) + ... = \ln 1$$

(d) Evaluate the sum:

$$\ln 1 = 0$$

Is it correct? If not, either briefly explain where the 'proof' goes wrong or give your own analysis of $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ (i.e., state whether it converges and to what).

(4) Do the following series converge or diverge? Explain why or why not (you don't need any of the 'tests' you learned in sections 11.3 or 11.4).

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{4^n + 3}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{3+2^{-n}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{2}\right)^n$

(e) A series $\sum_{n=1}^{\infty} a_n$ whose Nth partial sum is $s_N = \sum_{n=1}^{N} a_n = \frac{\ln(2N)}{\ln(N)}$

(f) $\sum_{n=1}^{\infty} \left(1 - \frac{2}{n}\right)^n$

(5) Consider the following **incorrect** 'proof' that $\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$ diverges.

(a) $\frac{1}{n(n-1)} = \frac{1}{n}\frac{1}{n-1}$

(b) The series $\sum_{n=3}^{\infty} \frac{1}{n}$ diverges by the p-test with $p = 1$.

(c) The series $\sum_{n=3}^{\infty} \frac{1}{n-1}$ has terms $\frac{1}{n-1} \geq \frac{1}{n}$, so by the comparison test, it diverges, too.

(d) The series diverges, because it's the product of two divergent series:

$$\sum_{n=3}^{\infty} \frac{1}{n(n-1)} = \left(\sum_{n=3}^{\infty} \frac{1}{n}\right)\left(\sum_{n=3}^{\infty} \frac{1}{n-1}\right) = \infty \times \infty = \infty.$$

Where did it go wrong? Which step is incorrect? Does the series $\sum_{n=3}^{\infty} \frac{1}{n(n-1)}$ converge or diverge? (Explain your answers)

(6) Do the following series converge or diverge? Explain why or why not (for these, you'll need to use the integral test, p-series, and comparison tests)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+2}}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1-n}{n2^n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n + \ln n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n + n^3}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{1 + \sin^2 n}{2^n}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{n + 2^n}{n2^n}$

(j) $\displaystyle\sum_{n=3}^{\infty} \frac{5n^3 - 3n}{n^2(n-2)(n^2+5)}$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$