# Study guide for Math 244 Midterm 1

### Monday, September 24

These are the sorts of questions you should know how to solve for the first midterm.

1. Let $\vec{F} = xy\vec{i} + e^{yz}\vec{j} + (2y + z)\vec{k}$. Calculate curl $\vec{F}$ and div $\vec{F}$.

2. Calculate the volume of solid in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$.

3. Calculate the volume of the solid bounded by the sphere $\rho = 3$ and the cone $\phi = -\pi/6$.

4. Calculate the volume of the surface bounded below by the surface $z = (x^2 + y^2)^2$ and above by the plane $z = 1$.

5. Calculate the surface area of the surface $z = x^2 + y$ with $-1 \leq x \leq 1$ and $0 \leq y \leq 2$.

6. Use the coordinate transform $x = u + 2v$, $y = 2v$ to calculate $\iint_R x + y \, dx \, dy$, where $R$ is the parallelogram with vertices $(0,0)$, $(1,2)$, $(1,4)$, and $(0,2)$.

7. Calculate $\int_C \vec{F} \cdot d\vec{r}$ where $C$ is the portion of the unit circle from the point $(1,0)$ to $(0,1)$ and $\vec{F} = (x^2, -y)$.

8. Use Green's theorem to calculate the counterclockwise circulation of $\vec{F} = (ye^x, xe^y)$ around the square with corners $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$.

9. Check that the vector field $\vec{F} = yz\cos(xy)\vec{i} + xz\cos(xy)\vec{j} + \sin(xy)\vec{k}$ is conservative.

10. Use Stokes' theorem to calculate
$$\iint_S \nabla \times (y\vec{i}) \cdot \vec{n} \, d\sigma,$$
where $S$ is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.

You are not expected to memorize Green's theorem and Stokes' theorem. You should also expect that a few problems will ask you to set up integrals but not actually compute them.